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On the Objectionable Character of certain Methods very generally adopted for the Determination and Division of Surplus in Life Assurance Companies. By CHARLES JELLICOE, *one of the Vice-Presidents of the Institute of Actuaries.*

[Read before the Institute 27th December, 1852, and ordered to be printed.]

IN a short paper read before the Institute at the early part of this year, after alluding to the erroneous methods commonly adopted for the determination and division of surplus in Life Assurance Companies, I intimated my intention to investigate the nature and character of those methods, and to endeavour to demonstrate the mischievous and injurious consequences resulting from them. In the present paper I propose to commence this undertaking, trusting that the undeniable importance of the subject will be considered a sufficient apology for my somewhat frequent introduction of it at these meetings.

The manner of determining surplus almost universally practised formerly is still, I regret to say, very general. That it should be persevered in as it is, after the somewhat serious consequences which it has been shown to entail have been pointed out, is rather surprising, and would lead to the inference that the *exposé* of its peculiar tendencies has not been understood. As I have on a former occasion explained, the great error which characterises the old method is the using for calculations a table constructed as the Northampton Table is; that is to say, in such a manner as that the premiums charged are directly deducible from the elements of it.

That this is an error has been, I think, sufficiently demonstrated in a paper read before the Institute a year or two back. On the present occasion, I shall content myself with simply showing the results arising from such a practice. For this purpose, let us suppose a Society charging the Northampton premiums, and making its calculations by that table, to have been in existence 40 years, and to consist of 4,000 members, each assured for £1,000 at the age of 30; but the whole body divided into four classes of 1,000 each, the duration of whose assurances is 10, 20, 30, and 40 years respectively. We will further suppose that the real rate of interest prevailing is 3 per cent., and the real mortality that called the "experience." Then, proceeding in the usual way, the liability under each of the four classes will come out as follows :—

	By Northampton Table.	By true Table.
	£	£
Value of £1,000,000, assured 10 years ..	141·978	145·947
„ „ „ 20 „ ..	276·590	305·089
„ „ „ 30 „ ..	425·051	480·059
„ „ „ 40 „ ..	594·938	648·611
	1·438·557	1·579·706

It will hence be seen that the "good old Northampton Table," as some have called it, is rather a dangerous one to trust to. In each of the foregoing cases it not only gives a false value, but, what is a much more serious consideration, an inadequate one;* the total liability, in the instance here supposed, being underestimated by no less a sum than £141·149. A similar result may be looked for whenever the rates of interest are the same, and the true mortality less than that of the factitious table. But, under any circumstances, the liability found by means of such a table can never be depended upon as the true measure of it; and hence it follows, that in Societies still using the Northampton and similarly constructed tables for this purpose, it is certain that the amount of the reserve made by them is not the right one, and that the sufficiency of it, generally speaking, is very questionable.

With the distinction between the true and false values of annuities, came in necessarily that between the true premium, or annual payment to provide for the sum assured merely, and such an one as included in addition the provision for extra contingencies. The doubts and difficulties which have hence arisen are quite extraordinary; and the confusion of ideas upon the subject which appears to prevail, more especially out of the Institute, serves at least to show the advantages to be derived from professional association, and the evils which such association is calculated to remove.

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* In the paper above alluded to it is shown that $A = \frac{1 + {}'A}{1 + \phi(1 + {}'A)} - 1$, where A is the factitious value of an annuity, such as that of the Northampton Table, and 'A the true value; and it is from this property that the discrepancies now adverted to take their origin. By "true value" and "true premium" I would beg to be understood, once for all, as meaning such value and such premium as result from a combination of rates of mortality and interest as nearly as practicable assimilating to the actually prevailing rates.

Under the old system, premiums known to be more than sufficient for the risk were valued as annuities of less than the true number of years' purchase; under the new system, they are valued at the true number of years' purchase—that is to say, credit is taken for their value as nearly as it can be assigned, and, it being well understood that such value comprehends not only the provision for the risk but for future surplus and expenses, it is equally well understood that the whole of it must be reserved, or that the provision in question will be intrenched upon. For the sake of greater precision, the component parts of this value may also be exhibited, and the due proportion assigned to each. To make the matter perfectly clear, I will give an instance of the result of each method, adopting in each case the supposition as to the amount of assurances in force I have heretofore made. According to the old system, the following will be the result:—

Liability.	£	*Assets.*	£
Value of sums assured	2,607,907	Value of premiums	1,169,352
		Difference, being the further sum required to provide for the risk and for other future contingencies	1,438,555
	£2,607,907		£2,607,907

The new method gives the following:—

Liabilities.	£	*Assets.*	£
Value of sums assured	2,490,810	Value of premiums	1,276,687
Value of marginal guaranty, or "guaranty fund"	365,583	Difference, being the further sum required to provide for the risk and for other future contingencies	1,579,706
	£2,856,393		£2,856,393

It will thus be seen, that by the much vaunted use of the Northampton Table every item is wrongly given; the value of the sums assured is too great, of the premiums too little—and, what is worse, the difference between the two comes out also too little. By the true method, each quantity is correctly represented, the value of the premiums, as before, including that portion of them required to provide for future extra contingencies, but the present value of this item appearing on the other side as the guaranty fund, viz., £365·583.*

Now it is the proportion of this "guaranty fund" to the "value of the premiums" which comprehends the all-important question of the real condition of any given Society. In the instance cited, it will be observed that the reserved fund is 28·6 per cent. of the value of the gross premiums; in other words, that of the £106,688 per annum, here supposed to be receivable by the Society, £30,488, or 28·6 per cent., worth in present value £365,583, is put aside as a reserve to meet the extra contingencies of future years. With the slender information which the other mode affords, it is not possible even to detect the fact that the "difference" is actually too little, much less to arrive at any idea of what amount is put aside over and above the provision for the mere amount of the risks.

Amongst other fallacies advanced on this subject, it has been urged that the valuing the gross premium is improper, inasmuch as it is an "anticipation of profits," and gives an exaggerated importance to the assets thus ascertained; now it is not only perfectly reasonable to proceed in this manner, but essentially necessary to do so. If we cannot assume that one portion of the premiums will be received, it is quite evident that we have no right to reckon upon any other portion; and hence upon this hypothesis the future premiums have no value at all. But the objection cannot be sustained! It is right to value them in this manner, and, as I have said, essential to do so: for if not, a Society charging very low premiums would show, *cæteris paribus*, the same assets as another charging very high ones, supposing both to value by the same table; that is, the "guaranty fund" would be omitted on both sides the account in both—the one Society nevertheless really possessing a much larger guaranty than the other.

But it is the omission itself which constitutes the principal evil;

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* I have adopted the term "guaranty fund" for want of a better. It is, of course, the value of the difference between the premiums charged and those required to provide the sums assured, and its proportion to the value of the gross premiums represents the "*bonus giving power*" of a Society. This power is, probably, as high as it ever need be when the ratio is as one to three—that is, when the fund in question is one-third of the value of the gross premiums. The less this proportion, the less, of course, the "bonus giving power." The severity of this test may not be understood very well by the public just at present, but there can be no doubt that it soon will be.

for whilst a certain degree of credit may be taken for such omission, it is scarcely possible to discover whether the quantity omitted is of importance or not. It is clearly necessary that its value be given, that it may be appreciated. Supposing, then, the rates at which the calculations are made to be known, and the calculations themselves correct, this per centage of the guaranty fund will at once disclose the real condition of any Life Assurance Society. It is a kind of *constant*, by which the relative position of such bodies may at any given epoch be fairly measured; and hence it is to the last degree important that it should be made to appear.[*]

There are other erroneous methods still practised in the determination of surplus, but they are of a minor character; and I shall, therefore, now proceed to consider such as have reference to a much wider field of operation, viz., the division or distribution of the surplus amongst the members. I have on former occasions touched theoretically upon this subject, and I think it has been sufficiently demonstrated that the true mode of dividing any surplus is in proportion to the amount at the prevailing rate of interest of the payments made by each contributor over and above those required for the risk.[†] I shall, therefore, in what follows, assume it to be so, and endeavour to show, by contrasting this true mode with other methods very commonly adopted, to what extent they are departures from the truth, and in what degree inequitable.

There is, probably, no system which has at the present day more general sanction than that which distributes the surplus in proportion to the excess which the amount of premiums at compound interest exhibits over the value of the assurance in each case.[‡] The author of this system seems to have thought, that as the total surplus, after deduction of the expenses, was the difference between the amount of all the premiums at compound interest and the value of all the assurances, the share of each would be found on the same principle. Had he deducted the amount of the true premium instead of the value of the assurance, he would have been right. He overlooked the fact that, beside the value of the assurance, a considerable portion of the premiums paid must have gone to meet the claims, and that, omitting this element, the surplus would be divided, not in the proportion in which each had contributed to form it, but in one compounded of the contribution to surplus and to the payment of claims. In short, the obvious consideration escaped him, that, had the premiums been barely sufficient for the risk, there could have been no surplus at all; and therefore, that whatever there was irrespective of expenses must have arisen from the premiums being more than sufficient for that object.

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[*] The advocates for maintaining the old system, who are so tenacious of introducing into the account the item above referred to, lest such introduction should lead to the appropriation of any part of it, appear to forget that there may be a very little margin as well as a very large one; and that, in their anxious desire to conceal the latter state of affairs, they quite keep out of sight the danger arising from the former.

[†] That is, the share of each contributor is made equal to $\phi a^n \cdot \frac{s}{\int . \phi a^n}$ adopting the same notation as that given on former occasions.

[‡] That is, the share of each contributor is made equal to $(pa^n - l) \cdot \frac{s}{\int . pa^n - \int . l}$.

Proceeding, then, in the manner inculcated by these two methods, and assuming the same data as before, let us contrast the results as they would appear at a division; the assurances participating in which we may suppose to have commenced at ages 20, 30, 40, and 50 respectively.

The following table shows this contrast as it would exhibit itself at what we may suppose to be—

The First Septennial Division.

Age when assurance effected.	Share of Surplus.	
	By true method.	By false method.
20	55·7	41·6
30	60·1	48·4
40	62·1	60·8
50	54·9	82·0

From the above it will be seen that some of the younger members are deprived of their fair share of profits, by the system under consideration, to the extent of about 25 per cent.; whilst their more fortunate seniors get an excess, in some cases, of not less than 50 per cent. beyond the sum they are properly entitled to. Those alone who commence their assurances in the middle period of life receive their due proportion; at either extreme, the inequitable operation of the system is at its highest point.

Now it is not a little remarkable, that the differences between the amount of premiums at interest (always taken from the origin as is, I believe, the usual practice), and the value of the policy, preserve for many years nearly the very same ratio (the number of years being, of course, the same in each case); and hence the foregoing table will truly represent what takes place at subsequent divisions, since the ratios of the true method are obviously always the same. Thus, at the third septennial division (that is to say, at the end of twenty-one years), the ratios will be as follows:—

Third Septennial Division.

Age when assurance effected.	Share of Surplus.	
	By true method.	By false method.
20	55·7	41·3
30	60·1	48·1
40	62·1	61·4
50	54·9	82·0

nearly identical, as will be seen, with those exhibited in the first; and thus we arrive at the conclusion that, at each septennial division, the same degree, or very nearly the same degree, of injustice is perpetrated by the erroneous method in question as at the first; that is to say, that the share of surplus, or bonus given to the persons who commence their assurances about the age of 50 and upwards, is more than twice as great as it ought to be, and that the deficiency thus created is made good by mulcting those who enter the Society at the earlier periods of life. These observations, it must be remembered, apply to a Society charging the Northampton premiums, as they are called, and making its calculations by the Northampton Table, and in which the prevailing rate of interest is the same, viz., three per cent., but the mortality similar to that called the "experience"; any change in these data will produce a corresponding one in the ratios above given, but none is likely to occur in practice which will altogether remedy the evils here shown to exist.

The next system which I propose to investigate is that which apportions the surplus amongst the members in the ratios indicated by the amount of their payments at compound interest.* In this, as in the instances previously noticed, the irregularities will take a particular character as variations occur in the data. As it will be useful to have the results in such a case, I will now suppose the real rates of mortality and interest to be the "experience" and three-and-one-half per cent. respectively, other things remaining as before. The ratios resulting from the two methods will then be as follows:—

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* That is, the share of each contributor is made equal to $pa^n . \frac{s}{\int . pa_n}$.

First Septennial Division.

Age when assurance effected.	Share of Surplus.	
	By true method.	By false method.
20	54·7	39·6
30	59·7	48·7
40	62·8	61·9
50	56·6	82·6

Here we see that, by the method now under discussion, the older lives get about 46 per cent. more than their due share, and that the youngest in the table loses nearly 28 per cent. of his. It will be observed that the ratios afforded by the true method vary very little from those before given, notwithstanding the change in the rate of interest from three to three-and-one-half per cent. These proportions, like those already adverted to, change very insignificantly as the number of years increases; and hence this method, like the foregoing one, gives the persons assured at the more advanced ages a very great advantage over those commencing their assurances at earlier periods of life. It is, perhaps, hardly necessary to mention, that the reason of the discrepancy in this case is, that the surplus is shared in proportion to the contribution to it, and to the amount of the payments for the risk, instead of in proportion to the former only.[*]

The third mode which I have to adduce, and with the investigation of which I shall conclude this paper, is one of more rare occurrence than those we have now discussed. It is, nevertheless, still practised by some Societies, and would seem to be based upon the idea that the allotment of surplus should be made in direct proportion to the amount of each person's interest in the funds of the Society. It does, in fact, distribute the surplus in proportion to the value of each policy respectively;[†] or, in other words, the Society is thus called upon to divide its profits in proportion to its liabilities—an arrangement which must be looked upon not only as

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[*] That is, the share of each contributor is made equal to

$$(p'+\phi)a^n \, . \, \frac{s}{\int . \, (p'+\phi)a^n} \text{ or } pa^n \, . \, \frac{s}{\int . \, pa^n} \, ;$$

and I may here add, that when the premiums charged are made up of the "true ones," increased by a per centage, this method would be correct, otherwise not.

[†] That is, the share of each contributor is made equal to $l \, . \, \frac{s}{\int . \, l}$.

purely arbitrary, but, to say the least, as having nothing whatever to justify it. The manner in which it operates will be seen by the following statement:—

First Septennial Division.

Age when Assurance effected.	Share of Surplus.	
	By true method.	By false method.
20	55·7	36·8
30	60·1	48·4
40	62·1	63·9
50	54·9	83·7

A comparison of this statement with those which have gone before, will show that the errors lie precisely in the same direction. It differs from them merely inasmuch as it exhibits those errors in a still more exaggerated shape. As in the former instances, the ratios do not vary very materially with the lapse of time. The following statement shows their relation at the end of twenty-one years, or, at what we may suppose to be the

Third Septennial Division.

Age when Assurance effected.	Share of Surplus.	
	By true method.	By false method.
20	55·7	37·9
30	60·1	48·9
40	62·1	62·9
50	54·9	83·1

rather, as it will be observed when compared with the foregoing, exhibiting a tendency in the right direction, albeit a very insignificant one.

On reviewing what has been said in reference to these three erroneous modes of division, the members cannot fail to be struck with the singularity of the circumstance, that although they apparently differ so much in principle, they should produce results so nearly similar. A little reflection, however, will show that they do not really differ so much in principle as at first they may seem to do. It is to be remembered, that the value of a policy always bears a certain relation to the amount of premiums paid upon it; and hence the differences between these quantities will have a tendency to preserve a similar relation.* Whether, therefore, a division be made in proportion to the amount of premiums paid, to the value of the policy, or to the difference between the two, the results will not be very widely different, although of course in each case at variance with the true and proper ones.

Before concluding, it may be worth while to observe the effect of these erroneous methods on the supposition of a given amount of additions having been made or become vested up to the present time. Let us suppose such amount to be £4,656,000 (a sum far below the truth, but assumed for the convenience of calculation merely), and let us further imagine it to have been so paid or become vested in respect of assurances effected at the ages before specified; the following table will exhibit what has been done with this sum, as contrasted with the manner in which it should have been appropriated, the several conditions being such as those I have before assumed.

Age when Assurances effected.	Additions paid or allotted.	Additions as they should have been paid or allotted.
	£	£
20	826,000	1,114,000
30	962,000	1,202,000
40	1,228,000	1,242,000
50	1,640,000	1,098,000

Here it will appear, upon our hypothesis, that a sum of about £530,000 is taken out of the pockets of the younger members, and handed over to the older ones, without anything, it may be, to justify such a proceeding. I do not for a moment mean to deny that under certain conditions these anomalies will sometimes altogether vanish, and the reasoning here mooted be inapplicable. I wish merely to draw attention to the subject generally, and to the necessity of carefully watching what the conditions in any particular case may be, and of adopting such methods of procedure as are suited to those conditions, and as are calculated, in the first instance, to determine the amount of surplus accurately, and thereupon to distribute it amongst the contributors in as rational and equitable a manner as possible.

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* See Euclid, book v., proposition 19.